A Mathematical Question

In an exercise, you were asked to show $g(f(x))$ is integrable when $f \in R[a,b]$ and $g \in C(\mathbb{R})$. A natural question is, is it true that $g(f(x)) \in R[a,b]$ when $f \in C[a,b]$ and g is integrable (over the range of f) ?

Some of you asked me this question after class. This is a good one. My immediate answer was it is no longer true. However, the construction of an example was not available. Here is one.

Let C be the famous Cantor set. It is known that it is a set of measure zero in $[0,1]$. We may modify the construction of the Cantor set to obtain a Cantor-like set E which has positive measure. There is a continuous, monotone increasing function which maps E bijectively to C, denote it by f. Let g be the characteristic function of C. Since g is discontinuous precisely at C which is of measure zero, by Lebesgue's Theorem it is integrable. But then the function $g(f(x))$ is discontinuous at E. By Lebesgue's Theorem it is not integrable.

Sorry for I fail to identity the student, so I post my answer here for him and others.